SHORT COMMUNICATIONS

State space approach for joint estimation of activity and attenuation map from PET emission sinograms*

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Accepted on October 30, 2006

Abstract Quantitative estimation of radioactivity map has important clinical implications for better diagnosis and understanding of cancers. Although attenuation map and activity map are usually treated sequentially, they can obviously benefit a great deal when the transmission data is missing. In this paper, we propose a novel scheme of simultaneously solving for attenuation map and activity distribution from emission sinograms. Our strategy combines the measurement model of PET, and the attenuation parameters are treated as random variables with known prior statistics. After the conversion to state space representation, the extended Kalman filtering procedures are adopted to linearize the equations and to provide the joint estimates in an approximate optimal sense. Experiments have been performed on both synthetic data to illustrate its abilities and benefits.

Keywords: positron emission tomography, attenuation correction, state space methods.

Positron emission tomography (PET) has been playing a more and more important role in clinical oncology and patient care in recent years. During the PET scan, some chemical compounds labelled with positron emitting isotopes are pre-injected into the human body to generate detectable gamma photon pairs through the positron annihilation events. By coincidence detecting photon pairs from true events and counted during the scan duration (the procedure called emission scan), an image indicating tissue metabolic activity could be reconstructed from the data.

Attenuation is the loss of true events due to scatter and absorption of photons when they pass through body tissue. As an attenuating medium, the body prevents a substantial fraction of photons reaching the detector primarily through Compton scattering[1]. Therefore, the image quality and quantitative accuracy of PET reconstruction is degraded. In clinical application, significant metabolic changes in tumor can be monitored by comparing the uptake values from pre- and post-treatment scans. These comparisons can only be made accurately on attenuation-corrected, quantitative PET images. Tissue radio-attenuation map is usually reconstructed based on the acquired transmission data by scanning the patient with a rotating external radionuclide source[2—4]. However, in addition to financial costs, the transmission scan will increase the radiation dosage and thus the risk for the subject. Further, it may be difficult for some patients to tolerate for near or over one hour scan at one time and thus leads to co-registration problem in the emission image reconstruction. It is thus the goal of many efforts to recover the attenuation map without performing transmission scan, including the use of a priori information of attenuation coefficients[5,6], and the attempt to derive the attenuation map directly from the measured emission data[7]. Recently, a maximum-likelihood framework for joint recovery of activity distribution and attenuation coefficient map was proposed[8]. The main advantage of the method is that it will provide a reasonable amount of information when the transmission data is uhavailable. Following this, we present a joint estimation approach to recovery the tissue attenuation and radioactivity distribution simultaneously from emission sinograms.

* Supported by National Program on Key Basic Research Projects (Grant No. 2003CB716100), the Hong Kong Research Grants Council (HKUST6151/03E), National Natural Science Foundation of China (60021201) and the Key Program of Sci & Tech Department of Zhejiang Province (2005C21019)

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With the PET imaging and measurement processes in state space representation, the attenuation parameters are treated as random variables with known prior statistics in our effort. In our current implementation, the extended Kalman filter (EKF) procedures are adopted to linearize the augmented state representation to provide the joint estimates in the minimum-mean-square-error sense (the imaging data undergoes Anscombe transformation first to be converted into Gaussian distribution).

1 Methodology

1.1 PET emission scan model

Assuming that the activity distribution and attenuation map are given by column vectors $\boldsymbol{x} = \{x_j \mid j = 1, \cdots, N\}$ and $\boldsymbol{\mu} = \{\mu_j \mid j = 1, \cdots, N\}$ accordingly, where N is the total number of image voxels, x_j the activity in voxel j and μ_j the linear attenuation coefficient in the same voxel. The survival probability from attenuation of a photon, which is equal to linear integration along the its path, can be written in a discrete form as

$$a_i = \exp\left(-\sum_j l_{ij}\mu_j\right) \tag{1}$$

where l_{ij} represents the effective intersection length along projection i. Thus the attenuation factor can be represented by a diagonal matrix $\boldsymbol{A}$, of which the diagonal element is $[A]_{ii} = a_i$.

Now we use d_{ij} to indicate the probability that a photon pair produced in voxel j reaches the front face of detector pair along projection i without attenuated, and a matrix $\boldsymbol{D}$, which is usually called system matrix, to contain all these probabilities. By making another matrix $\boldsymbol{L}$ contain all intersection length along every projection and in every voxel, a mathematical model of the PET emission measurement could be written as:

$$y_i = \exp\left(-\sum_j l_{ij}\mu_j\right)\left(\sum_j \mathrm{d}_{ij}x_j\right) + \text{noise} \tag{2}$$

or in matrix form

$$y = \boldsymbol{AD}x + \text{noise} = \mathrm{diag}(e^{-[L\mu]})Dx + \text{noise} \tag{3}$$

Note that the symbol diag (x) means a diagonal matrix expanded with all elements in a vector x as its diagonal elements.

1.2 Nonlinear state space representation

A state space representation used for PET imaging reconstruction[9] could be written as

$$z(t+1) = Cz(t) + v(t)$$
$$y(t) = \boldsymbol{D}z(t) + \omega(t)$$

Since the attenuation map has an exponential relation with emission sinogram data as in (3), by assuming the activity and the attenuation map in one measurement as static image series, we have a simplified state-space model with a nonlinear measurement equation as

$$z(t+1) = z(t) + v \tag{4}$$
$$y(t) = f(z(t)) + r \tag{5}$$

with a joint state variable

$$z(t) = \begin{bmatrix} x(t) \\ \mu(t) \end{bmatrix}$$

where v, r are zero-mean random vector variables with known covariance matrices $E[rr^{\mathrm{T}}] = \boldsymbol{R} > 0$, $E[vv^{\mathrm{T}}] = \boldsymbol{Q} > 0$. The nonlinear function in measurement equation has the form according to (3)

$$f(z(t)) = \mathrm{diag}(e^{[-L\mu(t)]_i}) \cdot \boldsymbol{D}x(t) \tag{6}$$

By solving $\hat{z}(t)$ from above nonlinear state space equation, the estimation of activity map $\hat{x}(t)$ and attenuation map $\hat{\mu}(t)$ can be obtained simultaneously.

2.3 Extended Kalman filter solution for joint estimation

An extended Kalman filter (EKF) approach[10] is adopted to estimate the state variable from the nonlinear stationary state-space system (4), (5). In order to apply the EKF, the nonlinear function f is recursively linearized around the most recent estimate of $z(t)$ by taking gradient of the transformation function at time t according to

$$\boldsymbol{H}(t) = \left.\frac{\partial f(z)}{\partial z^{\mathrm{T}}}\right|_{z = \hat{z}^-(t)} \tag{7}$$

The estimated value $\hat{z}(t)$ of vector $z(t)$ can be carried out by the following iterative process predication

$$\hat{z}^-(t+1) = \hat{z}(t) \tag{8}$$
$$\boldsymbol{P}^-(t+1) = \boldsymbol{P}(t) + \boldsymbol{Q} \tag{9}$$

and correction

$$\hat{z}(t) = z^-(t) + \boldsymbol{K}(t)[y(t) - f(\hat{z}^-(t))] \tag{10}$$

$$\boldsymbol{P}(t) = [\boldsymbol{I} - \boldsymbol{K}(t)\boldsymbol{H}(t)]\boldsymbol{P}^-(t) \tag{11}$$

where

$$\boldsymbol{K}(t) = \boldsymbol{P}^-(t)\boldsymbol{H}^{\mathrm{T}}(t)[\boldsymbol{H}(t)\boldsymbol{P}^-(t)\boldsymbol{H}^{\mathrm{T}}(t) + \boldsymbol{R}]^{-1}$$

The matrix $\boldsymbol{K}(t)$ is called Kalman gain matrix, which drives the improvement of $\hat{z}(t)$ and $\boldsymbol{P}$ in the correction step by weighting the new information coming from next system measurement $y(t)$ and the previously stored information in prediction step.

Here we treat the emission measurement as a stationary process that each time we get the same measurement result and with constant state variables to apply the state-space model on it. The initialization at time $t=0$ of the state vector $\boldsymbol{z}(0)$ and the relevant covariance matrix $\boldsymbol{P}(0)=E\{[z(0)-\hat{z}(0)][z(0)-\hat{z}(0)]^{\mathrm{T}}\}$ is given at the beginning of the iteration. The covariance of the measurement error R and system error Q is known and set to time invariant for the static measurement process.

2 Experimental results

The digital zubal thorax phantom with known radioactivity concentrations and attenuation coefficient distribution is used, as shown in Fig.1, to evaluate our method.

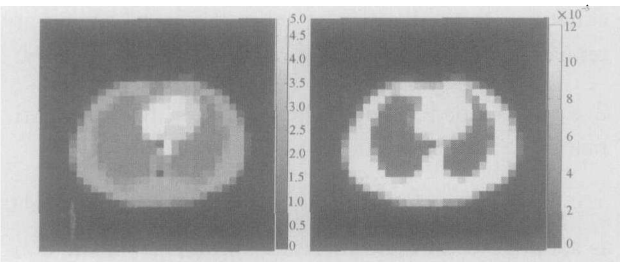

Fig. 1. Digital zubal thorax phantom for PET reconstruction experiments, activity distribution (left), attenuation map (right).

The resolution of the original image is 32×32 pixels, while 1024 projections over 180 degrees are simulated. To generate realistic data, we simulate the emission coincidence events considering tissue attenuation and detection probabilities, the measured sinogram y is created based on Eq. (3), where the noise mainly is 20 percents scatter events and 50 percents pre-corrected random events with a total counts at 20 M. The probability matrix and intersection length matrix are generated using the MATLAB tool box developed by J. Fessler and his students.

An EKF iteration for solving state space Eqs. (8)—(11) is applied to the simulated sinogram data after giving an initial estimates of activity by filtered backprojection (FBP) and a rough attenuation map as an a priori information. Other parameters, that the initial estimation error covariance P_0, the system error covariance matrix $\boldsymbol{Q}$ and the measurement error covariance matrixes $\boldsymbol{R}$, are initialized at the beginning of iteration.

The images of the activity map and attenuation map obtained by joint estimation and the activity map just by common FBP and EM-ML method are shown in Fig. 2. For comparison of the recovered activity maps, we also use the popular EM-ML strategy to recover the activity parameters with the exact known attenuation map. Since its attenuation map is exactly the same as the ones used for data generation, the EM-ML strategy with known attenuation values (AC-EM-ML) produces the closest results to the ground truth among the four experiments. The horizontal profiles of the 19th row through the activity and attenuation distribution images are plotted in Fig. 3. It is also shown that the joint estimation result of activity distribution is much better than FBP and EM-ML in terms of quantitation.

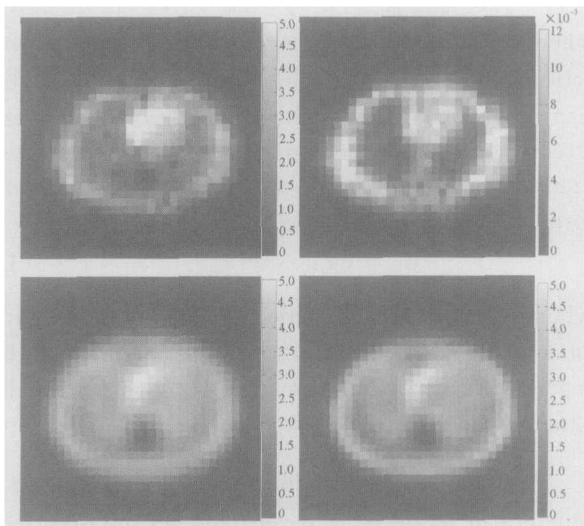

Fig. 2. Activity map (top left) and attenuation map (top right) images (Zubal thorax phantom) recovered by joint estimation, and activity map reconstructed by FBP (bottom left) and EM-ML (bottom right).

3 Conclusions

In this paper, we have presented a joint estimation of both activity distribution and attenuation map by a state space approach from the emission sinograms alone. Constructing the emission scan system and measurement equations with a consideration of attenuation effect, we rely on state space technique and use extended Kalman filtering iteration to generate joint estimates of PET images and tissue attenuation. The joint estimation is particularly meaningful for quantitative PET imaging neglecting the transmission scan. Analysis and experiment results with a digital Zubal thorax phantom data demonstrate the power of the new proposed method.

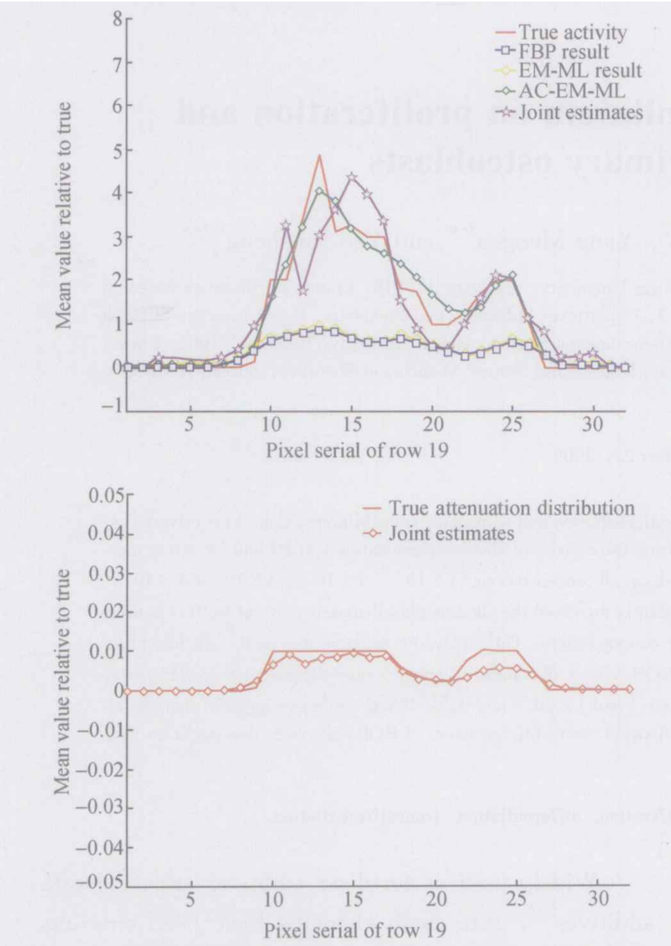

Fig. 3. Horizontal profiles of the 19th row through activity maps (top) by different estimators and attenuation map by join estimation (bottom).

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